## Stringent constraint on the scalar-neutrino coupling constant from quintessential cosmology

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## Abstract

An extremely light ( $m_{\phi} \ll 10^{-33}$  eV), slowly-varying scalar field  $\phi$  (quintessence) with a potential energy density as large as 60% of the critical density has been proposed as the origin of the accelerated expansion of the Universe at present. The interaction of this smoothly distributed component with another predominately smooth component, the cosmic neutrino background, is studied. The slow-roll approximation for generic  $\phi$  potentials may then be used to obtain a limit on the scalar-neutrino coupling constant, found to be many orders of magnitude more stringent than the limits set by observations of neutrinos from SN 1987A. In addition, if quintessential theory allows for a violation of the equivalence principle in the sector of neutrinos, the current solar neutrino data can probe such a violation at the  $10^{-10}$  level.

There are now increasing indications for a spatially flat Universe ( $\Omega_0 \equiv \rho_{total}/\rho_{crit} = 1$ ), in which a large fraction of the present energy density comes from a smooth component with negative pressure that is causing the accelerated expansion of the Universe. The simplest and at the same time the oldest known candidate providing the necessary negative pressure is a non-vanishing cosmological constant. However, other possibilities with similar properties have recently been proposed, including a dynamical, slowly-rolling, spatially inhomogeneous scalar field component, named quintessence [1]. The basic idea of quintessence is that of a classically unstable field that is rolling toward its true minimum, which is presumed to vanish. From a theoretical viewpoint, although this does not avoid the cosmological constant problem, it still supports a widespread belief that when the problem is properly understood, the final answer will be zero. From an observational viewpoint, it seems that the best-fit models [2] are those of quintessence with an effective equation-of-state  $\omega > -1$ , rather than the limiting case of the cosmological constant with  $\omega = -1$ .

Beside the cosmological constant problem, there is another problem with the rolling scalar field scenario-the initial conditions problem- one should answer why the energy density of the scalar field  $\Omega_{\phi}$  and the matter energy density  $\Omega_{m}$  are of the same order of magnitude today as we know that the energy density of the scalar field generally decreases more slowly than that of matter. Recently, the notion of cosmological "tracker fields" has been introduced in certain models of quintessence [3] to explain why we live in a special era where the two densities nearly coincide.

Another set of difficulties besetting the above scenario for quintessence has to do with the lightness of quintessence as well as with the flatness conditions obeyed by the potential  $V(\phi)$  in any realistic model of quintessence. In the first case [4], since the scalar field  $\phi$  is very light (or massless) and can mediate long-range forces, it must be subject to the constraints derived from the observational limits on a fifth force. In the latter case [5], the flatness conditions serve to restrict any additional parameter (other than generic non-perturbative ones) in  $V(\phi)$ . The result of the analysis [4] shows that only a moderate suppression of a few observable interactions of quintessence with the fields of the standard model is required, whereas the analysis [5] shows that high-degree of fine-tuning of certain parameters in  $V(\phi)$  is required, even in the context of supersymmetry.

In the present Letter, we combine the long-range phenomenon of quintessence with the flatness conditions for  $V(\phi)$ . Firstly, we calculate the shifted mass of  $\phi$  which in each point of space results from the interaction with the cosmic neutrino background, and then apply the flatness conditions to restrict a scalar-neutrino coupling constant, on which no severe constraints exist. Then we proceed by employing a mechanism for generation of neutrino oscillations similar to that developed in [6], in case an underlying quintessential theory allows for a violation of the equivalence principle (VEP) through a non-universal scalar-neutrino coupling. We find that the current solar neutrino data can then probe a VEP, and compare its upper limit with the most restrictive limit for ordinary matter [7] as well as the limit on neutrinos obtained from SN 1987A [8].

We set the stage by writing down the fundamental equations governing the above scenario for quintessence. If, for the sake of simplicity, we assume that the total energy density has the critical value, then

$$V(\phi) \sim (3 \times 10^{-3} \text{ eV})^4$$
, (1)

where the numerical value in (1) is the present energy density  $3M_P^2H_0^2$ . Here  $M_P \equiv M_{Planck}/\sqrt{8\pi} = 2.4 \times 10^{18}$  GeV is the reduced Planck scale and  $H_0$  is the present value of the Hubble parameter. The effective equation of state for this component is very negative:

$$\omega \equiv \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \lesssim -1/3 , \qquad (2)$$

and its equation of motion is given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 , \qquad (3)$$

where  $V'(\phi)$  is the derivative of V with respect to  $\phi$ . In order to provide for negative pressure, quintessence should satisfy the slow-roll condition,  $3H\dot{\phi} = -V'(\phi)$ , and the necessary conditions for the slow-roll approximation to hold are the flatness conditions for  $V(\phi)$ ,

$$M_P |V'/V| \ll 1 , \qquad (4)$$

$$M_P^2 |V''/V| \ll 1$$
 (5)

The application of (4) and (5) to a generic non-perturbative part in V (we may consider a potential of inverse-power,  $V(\phi) = M^{4+\alpha}\phi^{-\alpha}$ , as an example of the tracker field, where M is a parameter [9]) together with the condition that  $\Omega_{\phi}$  is beginning to dominate just today, gives  $\phi \sim M_P$  [3,5]. The same for the bare mass term in  $V(\phi)$  gives  $m_{\phi} \ll 10^{-33}$ . Note that for the mass term in  $V(\phi)$  we need consider only the second flatness condition (5), provided that  $\phi \sim M_P$ .

Let us now suppose that quintessence couples to neutrinos with a Yukawa strength  $g_{\nu}$  (the vacuum mass term for neutrinos is of the Dirac type), and consider its interaction with the background neutrinos at an effective temperature  $T_{\nu} \simeq 2K$ . We shall always treat the neutrino component as a smooth one, since at present only 10-20% of the dark matter neutrinos are in galactic halos while the rest are distributed more smoothly, as suggested by numerical simulations [10] even for the mass for neutrinos in the range of tens of an eV. In addition, in the light of recent observation of atmospheric neutrino oscillations (and hence neutrino mass) at Super-Kamiokande [11] we consider either of two possibilities for neutrino masses that are consistent with the current data: the one where the neutrino masses are hierarchical with the highest mass eigenvalue  $\sim 0.1$  eV, and also the case where some neutrino masses are nearly degenerate and larger than  $\sim 0.1$  eV.

Here we would like to stress that the interaction just mentioned above would unavoidably induce an effective mass squared  $m_{\phi}^2 + \Pi_{\phi}(0)$  in  $V(\phi)$  [12], where  $\Pi_{\phi}$  is the scalar self energy at finite temperature. In this problem we take the infrared limit which is obtained by setting the zeroth component of the external momentum to zero and taking the limit that spatial components approach zero, i.e.,  $\Pi_{\phi}(k_0 = 0, \mathbf{k} \to \mathbf{0}) \equiv \Pi_{\phi}(0)$ .

For neutrino components with  $m_{\nu} \ll T_{\nu}$ , we find, by applying the real-time version of Thermal Field Theory [13], at the one loop level that

$$\Pi_{\phi}(0) \equiv \Pi_{\phi}^{htl}(k_0, \mathbf{k}) \simeq \frac{g_{\nu}^2 T_{\nu}^2}{12} .$$
(6)

In the opposite and more realistic case,  $m_{\nu} \gg T_{\nu}$ , we find

of out of equilibrium thermal field theories.

$$\Pi_{\phi}(0) \simeq -0.07 \ g_{\nu}^2 \ m_{\nu} \ T_{\nu} \ , \tag{7}$$

where the parameters in Eqs.(6) and (7) refer to the heaviest neutrino from the background. Concerning Eqs. (6) and (7), a few technical remarks as well as additional illuminations are in order. First note that, in contrast to gauge theories, the calculation of the scalar self energy in the hard thermal loop approximation [14] shows a cancellation of the momentum-dependent terms, leading to the simple result, Eq.(6). Furthermore, the scalar self energy shows no imaginary part corresponding to Landau damping for all values of the momentum and the energy. In addition, Eq.(7) represents a specific non-equilibrium situation where massive neutrinos, which are nonrelativistic today, still need to be assigned a distribution function relevant for massless particles, as their total number density is fixed at about 100 cm<sup>-3</sup> in the uniform non-clustered background. This sort of non-equilibrium would unavoidably induce ill-defined pinch singularities at two-loop order [15] - a common feature

With respect to the sign in Eq.(7), one should not be overmuch surprised by the appearance of a "thermal tachyon". In contrast to scalar and gauge theories, where the mass squared generated by thermal fluctuations is always found to be positive, this is not necessarily true in a theory having interactions which are universally attractive, signaling that a thermal distribution just tends to collapse upon itself. Beside gravity, scalar theories with cubic interactions in six dimensions are such another example [16]. This feature however does not show up in Eq.(6) because of free streaming of relativistic background particles  $(m_{\nu} \ll T_{\nu})$ ; on the other hand Eq.(7) is closely related to the scalar contribution to the usual Jeans mass. Finally, we have taken the number of neutrino degrees of freedom to be equal 2 in the above equations while the chemical potential has been set to zero, as is probably the case for cosmological neutrinos.

Assuming no fine-tuned cancellations between various contributions to the slope of V, we are now in position, by applying the flatness condition as given by Eq.(5) directly to Eqs.(6) and (7), to set a limit on the scalar-neutrino coupling constant as

$$g_{\nu} \ll 10^{-28} \qquad (m_{\nu} \ll T_{\nu}) \,, \tag{8}$$

$$g_{\nu} \ll 4 \times 10^{-30} \left(\frac{0.07 \text{ eV}}{m_{\nu}}\right)^{1/2} \qquad (m_{\nu} \gg T_{\nu}) .$$
 (9)

The limits (8) <sup>1</sup> and (9) are to be compared with the most stringent limits on  $g_{\nu}$ , that is with those set by the observations of neutrinos from SN 1987A. By making a claim of absence of large scattering of supernova neutrinos from dark matter neutrinos, one obtains,  $g_{\nu} < 10^{-3}$  [17]. Moreover we show that the powerful bound as given by Eq.(9), for neutrinos

<sup>&</sup>lt;sup>1</sup>It is to be noted however that even better limit in the case where the cosmic neutrino background remains relativistic today can be obtained by considering the neutrino mass generated by the VEV of  $\phi$ ,  $\delta m_{\nu} = g_{\nu}\phi$ . The requirement  $\delta m_{\nu} \ll T_{\nu}$  then gives  $g_{\nu} \ll 10^{-31}$ .

having masses in the eV range, is even more restrictive than the corresponding limit for ordinary matter coming from conventional solar-system gravity experiments. The present experimental data give upper limits of order  $10^{-3}$  for a possible admixture of a scalar component to the relativistic gravitational interaction,  $\beta_{ext}^2 < 10^{-3}$  [18]. By adjusting  $g_{\nu}$  to be of gravitational origin only,  $\beta_{\nu} \equiv \sqrt{2} M_P(g_{\nu}/m_{\nu})$ , one obtains from Eq.(9) that

$$\beta_{\nu}^2 \ll 4 \times 10^{-6} \left(\frac{\text{eV}}{m_{\nu}}\right)^3$$
 (10)

It is to be noted however that for  $m_{\nu} \sim 1 \text{ eV}$ , Eq.(10) represents a moderate fine-tuning in  $V(\phi)$ . Indeed, from a traditional viewpoint, the expected values for the  $\beta_{\nu}$  are of order of unity since they represent interactions at the Planck scale. The possibility to suppress such a coupling by imposing symmetries is viable only in pseudo-Goldstone boson models of quintessence [4]. Here we give two possible solutions to the fine-tuning problem. The first solution is in agreement with the fact that the current data favor models with a cosmological constant over the mixed cold + hot dark matter models. In this respect, the presence of hot dark matter is no longer necessary (and eV neutrinos are not needed to provide this component) [19]. If we set  $m_{\nu} \sim 0.04 \text{ eV}$ , a value consistent with the Super-Kamiokande experiment, then  $\beta_{\nu} \ll 0.3$ , and sure enough there are ways to achieve this value without suppression by some symmetry. The second solution is a least coupling principle introduced originally by Damour and Polyakov [20] in string theory. It is based on a mechanism which provides that a (universal) coupling of the scalar (the string dilaton field in their example) with the rest of the world, being dependent on the VEV of  $\phi$  and hence time dependent, has a minimum close to the present value of the  $\phi$ 's VEV. Therefore, the present-day value for  $\beta_{\nu}$  can naturally be much less than unity. We have to assume however that the mechanism is also operative for quintessence since in their paper Damour and Polyakov dealt with the string dilaton, which, as a recent analysis [21] shows, cannot provide us with the negative equation of state, and therefore is useless for the dynamical component of quintessence.

In the rest of the paper we shall be concerned with the case where  $\beta$ 's from the neutrino sector are not universal but rather species-dependent, thereby violating the equivalence principle. Specifically this means that  $\beta_{\nu} \to \beta_{\nu_i}$ , where i is the i-type neutrino. This is just a basic ingredient of the scenario [20], in which the scalar may remain massless in the low-energy world and violates the equivalence principle. By inducing a nonzero mass squared difference for neutrinos within a medium, such exotic interactions may produce neutrino oscillations even for degenerate-in-mass neutrinos [6].

Let us now consider the case indicated in the foot-note, where a nonzero mass squared difference for neutrinos is due to the VEP of quintessence. By sticking with the degenerate-in-mass neutrinos, we find  $\Delta m^2$  for the oscillatory neutrinos 1 and 2 with the degenerate mass  $m_0$  as  $\Delta m^2 \simeq 2m_0\phi\Delta g_{\nu}$ , where  $\Delta g_{\nu} \equiv g_{\nu_2} - g_{\nu_1}$ . By setting  $\Delta m^2 \simeq 10^{-10}$  eV as to explain the solar neutrino data via oscillation in vacuum, one finds that current solar neutrino data probe the quintessential scenario VEP at the level

$$\Delta\beta \simeq 10^{-10} \left(\frac{\text{eV}}{m_0}\right)^2 \,. \tag{11}$$

One finds for neutrinos of mass  $\sim 1$  eV (such a model for neutrino masses where the three known neutrinos have nearly the same mass, of about  $\sim 1$  eV, was presented in Ref. [22])

that the right-handed side of (11) is not as good as the most severe limit for ordinary matter [7]. It is however better than the limit obtained by comparing neutrinos with antineutrinos from SN 1987A [8].

The limit (11) should not be confused with those obtained in Ref. [23] as there VEP is due to a non-universal tensor neutrino-gravity coupling, whereas here VEP arises due to a breakdown of universality in the coupling strength between the spin-0 particles and the neutrinos. One can also show that the above bounds remain unchanged in the case in which the vacuum mass terms for neutrinos are of the Majorana type.

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